

Stable thin-shell wormholes with a Chaplygin gas in Einstein-Maxwell-Gauss-Bonnet gravity

Z. Amirabi* and M. Halilsoy†

Department of Physics, Eastern Mediterranean University, G. Magusa, north Cyprus, Mersin 10 - Turkey.

We study thin-shell wormholes in Einstein-Maxwell-Gauss-Bonnet(EMGB) gravity whose equation of state obeys that of a generalized Chaplygin gas. For specific parameters such a wormhole becomes stable against radial perturbations whereas its energy density remains negative.

Among other aspects the foremost challenging problems related to thin-shell wormholes [1] are, *i*) positivity of energy density, and *ii*) stability against symmetry preserving perturbations. To overcome these problems recently there have been various attempts in Einstein-Gauss-Bonnet (EGB) gravity with Maxwell and Yang-Mills sources [2]. Specifically, with the negative Gauss-Bonnet (GB) parameter ($\alpha < 0$) we obtained stable thin-shell wormholes, obeying a linear equation of state, against radial perturbations [1, 2]. By linear equation of state it is meant that the energy density (σ) and surface pressure p satisfy a linear relation. To respond the other challenge, however, i.e. the positivity of the energy density ($\sigma > 0$), we maintain still a cautious optimism. To be realistic, only in the case of Einstein-Yang-Mills-Gauss-Bonnet (EYMGB) theory and in a finely-tuned narrow band of parameters we were able to beat both of the above stated challenges [2]. Our stability analysis with the unfortunate negative energy density was extended further to cover non-asymptotically flat (NAF) dilatonic solutions [3].

In this Brief Report we show that stability analysis of thin-shell wormholes extends to the case of a generalized Chaplygin gas which has already been considered within the context of Einstein-Maxwell thin-shells wormholes [4]. Due to the accelerated expansion of our universe a repulsive effect of a Chaplygin gas has been considered widely in recent times. From the same token therefore it would be interesting to see how a generalized Chaplygin gas supports a thin-shell wormhole against radial perturbations in Gauss-Bonnet (GB) gravity. For this purpose we perturb the thin-shell radially and reduce the equation into a particle in a potential well problem with zero total energy. The stability amounts to the determination of the negative domain for the potential. We obtain plots that provides us such physical regions indicating stable wormholes. For technical reasons we restrict ourselves only to the 5-dimensional plots.

The d -dimensional Einstein - Maxwell - Gauss - Bonnet (EMGB) action without cosmological constant

$$S = \frac{1}{16\pi G} \int \sqrt{|g|} d^d x \left(R + \alpha \mathcal{L}_{GB} - \frac{1}{4} \mathcal{F} \right). \quad (1)$$

where G is the d -dimensional Newton constant, $\mathcal{F} = F_{\mu\nu} F^{\mu\nu}$ is the Maxwell invariant and α is the Gauss - Bonnet(GB) parameter with Lagrangian

$$\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \quad (2)$$

Variation of S with respect to $g_{\mu\nu}$ yields the (EMGB) field equations,

$$G_{\mu\nu} + 2\alpha H_{\mu\nu} = T_{\mu\nu} \quad (3)$$

in which $H_{\mu\nu}$ and $T_{\mu\nu}$ are given by

$$H_{\mu\nu} = 2(-R_{\mu}^{\sigma\kappa\tau} R_{\nu\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma} R^{\rho\sigma} - 2R_{\mu\sigma} R^{\sigma}_{\nu} + R R_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} L_{GB}, \quad (4)$$

$$T_{\mu\nu} = F_{\mu\alpha} F_{\nu}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}. \quad (5)$$

Our static spherically symmetric metric ansatz will be

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \quad (6)$$

*Electronic address: zahra.amirabi@emu.edu.tr

†Electronic address: mustafa.halilsoy@emu.edu.tr

in which

$$d\Omega_{d-2}^2 = d\theta_1^2 + \sum_{i=2}^{d-2} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2 \quad (7)$$

$$0 \leq \theta_{d-2} \leq 2\pi, 0 \leq \theta_i \leq \pi, 1 \leq i \leq d-3$$

and $f(r)$ is to be found.

Construction of the thin - shell wormhole in the static spherically symmetric spacetime follows the standard procedure used before [1-3]. In this method we consider two copies $\mathcal{M}_{1,2}$ of the spacetime

$$\mathcal{M}_{1,2} = \{(t, r, \theta_1, \dots, \theta_{d-2}) | r \geq a, a > r_h\} \quad (8)$$

which are egotistically incomplete manifolds whose boundaries are given by the following timelike hypersurface

$$\Sigma_{1,2} = \{(t, r, \theta_1, \dots, \theta_{d-2}) | F(r) = r - a = 0, a > r_h\}. \quad (9)$$

By identifying the above hypersurfaces on $r = a$ one gets a geodesically complete manifold $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$.

We introduce the induced coordinates on the wormhole $\xi^a = (\tau, \theta_1, \theta_2, \dots)$ - with τ the proper time - in terms of the original bulk coordinates $x^\gamma = (t, r, \theta_1, \dots, \theta_{d-2})$. Further to the Israel junction conditions [5], the generalized Darmois-Israel boundary conditions [6], are chosen for the case of (EMGB) modified gravity. The latter conditions on Σ take the form

$$2\langle K_{ab} - Kh_{ab} \rangle + 4\alpha \langle 3J_{ab} - Jh_{ab} + 2P_{acdb}K^{cd} \rangle = -\kappa^2 S_{ab}, \quad (10)$$

in which $\langle \cdot \rangle$ stands for a jump across the hypersurface $\Sigma = \Sigma_1 = \Sigma_2$, $h_{ab} = g_{ab} - n_a n_b$ is the induced metric on Σ with normal vector n_a and $S_a^b = \text{diag}(\sigma, p_{\theta_1}, p_{\theta_2}, \dots)$ is the energy momentum tensor on the thin shell. Therein the extrinsic curvature K_{ab}^\pm (with trace K) is defined as

$$K_{ab}^\pm = -n_c^\pm \left(\frac{\partial^2 x^c}{\partial \xi^a \partial \xi^b} + \Gamma_{mn}^c \frac{\partial x^m}{\partial \xi^a} \frac{\partial x^n}{\partial \xi^b} \right)_{r=a}. \quad (11)$$

The divergence - free part of the Riemann tensor P_{abcd} and the tensor J_{ab} (with trace J) are given also by

$$P_{abcd} = R_{abcd} + (R_{bc}h_{da} - R_{bd}h_{ca}) - (R_{ac}h_{db} - R_{ad}h_{cb}) + \frac{1}{2}R(h_{ac}h_{db} - h_{ad}h_{cb}), \quad (12)$$

$$J_{ab} = \frac{1}{3} [2KK_{ac}K_b^c + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{ab} - K^2K_{ab}]. \quad (13)$$

The black hole solution of the EMGB field equations (with $\Lambda = 0$) is given and with $\tilde{\alpha} = (d-3)(d-4)\alpha$ [7]

$$f_\pm(r) = 1 + \frac{r^2}{2\tilde{\alpha}} \left(1 \pm \sqrt{1 + 4\tilde{\alpha} \left(\frac{2M}{8\pi r^{d-1}} - \frac{Q^2}{2(d-2)(d-3)r^{2(d-2)}} \right)} \right) \quad (14)$$

in which M is an integration constant related to the ADM mass of the BH and Q is the electric charge of the BH. The corresponding electric field 2-form is given by

$$\mathbf{F} = \frac{Q}{r^{2(d-2)}} dt \wedge dr. \quad (15)$$

The components of energy momentum tensor on the thin shell are

$$\sigma = -S_\tau^\tau = -\frac{\Delta(d-2)}{8\pi} \left[\frac{2}{a} - \frac{4\tilde{\alpha}}{3a^3} (\Delta^2 - 3(1 + \dot{a}^2)) \right], \quad (16)$$

$$S_{\theta_i}^{\theta_i} = p = \frac{1}{8\pi} \left\{ \frac{2(d-3)\Delta}{a} + \frac{2\ell}{\Delta} - \frac{4\tilde{\alpha}}{3a^2} \left[3\ell\Delta - \frac{3\ell}{\Delta} (1 + \dot{a}^2) + \frac{\Delta^3}{a} (d-5) - \frac{6\Delta}{a} \left(a\ddot{a} + \frac{d-5}{2} (1 + \dot{a}^2) \right) \right] \right\}, \quad (17)$$

in which $\ell = \ddot{a} + f'_\pm(a)/2$, $\Delta = \sqrt{f_\pm(a) + \dot{a}^2}$ and while a 'dot' implies derivative with respect to the proper time τ a 'prime' denotes differentiation with respect to the argument of the function. These expressions pertain to the static configuration if we consider $a = a_0 = \text{constant}$ and therefore

$$\sigma_0 = -\frac{\sqrt{f_\pm(a_0)}(d-2)}{8\pi} \left[\frac{2}{a_0} - \frac{4\tilde{\alpha}}{3a_0^3} (f_\pm(a_0) - 3) \right], \quad (18)$$

$$p_0 = \frac{\sqrt{f_\pm(a_0)}}{8\pi} \left\{ \frac{2(d-3)}{a_0} + \frac{f'_\pm(a_0)}{f_\pm(a_0)} - \frac{4\tilde{\alpha}}{3a_0^2} \left[\frac{3}{2} f'_\pm(a_0) - \frac{3f'_\pm(a_0)}{2f_\pm(a_0)} + (d-5) \left(\frac{f_\pm(a_0) - 3}{a_0} \right) \right] \right\}. \quad (19)$$

We add also that in the case of a dynamic throat the conservation equation amounts to

$$\frac{d}{d\tau} (\sigma a^{(d-2)}) + p \frac{d}{d\tau} (a^{(d-2)}) = 0. \quad (20)$$

Our aim in the sequel is to perturb the throat of the thin shell wormhole radially around the equilibrium radius a_0 . To do this, we assume that the equation of state is in the form of a generalized Chaplyng gas [4], i.e.,

$$p = \left(\frac{\sigma_0}{\sigma} \right)^\nu p_0 \quad (21)$$

in which $\nu \in [0, 1]$ is a free parameter and σ_0/p_0 correspond to σ/p at the equilibrium radius a_0 . We plug in the latter expression into the conservation energy equation (20) to find a closed form for the dynamic tension on the thin shell after perturbation as follows

$$\sigma(a) = \sigma_0 \left[\left(\frac{a_0}{a} \right)^{(1+\nu)(d-2)} + \frac{p_0}{(d-2)\sigma_0} \left(\left(\frac{a_0}{a} \right)^{(1+\nu)(d-2)} - 1 \right) \right]^{\frac{1}{1+\nu}}. \quad (22)$$

Equating this with the one found in Eq. (16), i.e.,

$$-\frac{\sqrt{f_\pm(a) + \dot{a}^2}(d-2)}{8\pi} \left[\frac{2}{a} - \frac{4\tilde{\alpha}}{3a^3} (f_\pm(a) - 2\dot{a}^2 - 3) \right] = \quad (23)$$

$$\sigma_0 \left[\left(\frac{a_0}{a} \right)^{(1+\nu)(d-2)} + \frac{p_0}{(d-2)\sigma_0} \left(\left(\frac{a_0}{a} \right)^{(1+\nu)(d-2)} - 1 \right) \right]^{\frac{1}{1+\nu}}. \quad (24)$$

one finds a particle-like equation of motion which describes the behavior of the throat as

$$\dot{a}^2 + V(a) = 0, \quad (25)$$

The intricate potential here is given by

$$V(a) = f_\pm(a) - \left(\left[\sqrt{\mathbb{A}^2 + \mathbb{B}^3} - \mathbb{A} \right]^{1/3} - \frac{\mathbb{B}}{\left[\sqrt{\mathbb{A}^2 + \mathbb{B}^3} - \mathbb{A} \right]^{1/3}} \right)^2 \quad (26)$$

where

$$\mathbb{A} = \frac{3\pi a^3 \sigma_0}{2\tilde{\alpha}(d-2)} \left[\left(\frac{a_0}{a} \right)^{(1+\nu)(d-2)} + \frac{p_0}{(d-2)\sigma_0} \left(\left(\frac{a_0}{a} \right)^{(1+\nu)(d-2)} - 1 \right) \right]^{\frac{1}{1+\nu}}, \quad (27)$$

$$\mathbb{B} = \frac{a^2}{4\tilde{\alpha}} + \frac{1 - f_\pm(a)}{2}. \quad (28)$$

Having a stable wormhole imposes $V(a)$ to get negative values. Therefore we search for the regions in our parameters to satisfy this constraint on $V(a)$.

Fig. 1 represents a 5-dimensional plot for a black hole with two horizons. For the parameters shown a projection into the $\nu - a_0$ plane is given for a stable region, i.e. $V(a_0) < 0$. Fig. 2 also depicts a similar plot which does not correspond to a black hole case. Both pictures reveal the fact that in the stability domain we have $\sigma_0 < 0$.

In conclusion, for a generalized Chaplyng gas obeying the equation of state $p = \left(\frac{\sigma_0}{\sigma} \right)^\nu p_0$, we have found stable regions within physically acceptable range of parameters in EMGB gravity. The energy-density, however, turns out to be negative to suppress such a thin-shell wormhole as a prominent candidate.

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Figure Captions

Fig. 1: The 5-dimensional plot of stability region $V(a_0) < 0$, (from Eq. (26)) for the chosen constants versus the parameters ν and a_0 . The figures of $f(a_0)$ and $\sigma_0(a_0)$ are also given.

Fig. 2: The stability region ($d = 5$, again), in this case covers a larger radial domain but a restricted range for the Chaplygin parameter ν . For the chosen parameters $f(a_0)$ doesn't attain zero (or horizon). The stability region has again $\sigma_0 < 0$ as in Fig. 1.

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